FINAL TEST JANUARY 2011

Course : Matrix Algebra

Guideline of the answers.

1. $\begin{vmatrix} -1 & 1 & -1 & 1 \\ x & 2 & 1 & -3 \\ -3 & 1 & 2 & 0 \\ -2 & 7 & 0 & 2 \end{vmatrix} = -84 \rightarrow x = 0.$

2. $\begin{cases} -2x_1 + 2x_2 + x_3 = -1 \\ x_1 + x_2 - 2x_3 = 3 \\ 4x_1 - 3x_2 - x_3 = 5 \end{cases} \rightarrow \boldsymbol{A} = \begin{pmatrix} -2 & 2 & 1 \\ 1 & 1 & -2 \\ 4 & -3 & -1 \end{pmatrix} \rightarrow \boldsymbol{A}^{-1} = \frac{1}{-7}\begin{pmatrix} -7 & -1 & -5 \\ -7 & -2 & -3 \\ -7 & 2 & -4 \end{pmatrix}$

$$\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{G} = \frac{1}{-7}\begin{pmatrix} -7 & -1 & -5 \\ -7 & -2 & -3 \\ -7 & 2 & -4 \end{pmatrix}\begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \rightarrow x_1 = 3,\ x_2 = 2, \text{ and } x_3 = 1.$$

3. $\begin{cases} x + 2y + z - s + 3w = 0 \\ 2x + 4y + 2z - s + 7w = 0 \\ x + 2y + 2z + s + 2w = 0 \end{cases} \rightarrow \begin{pmatrix} 1 & 2 & 1 & -1 & 3 \\ 2 & 4 & 2 & -1 & 7 \\ 1 & 2 & 2 & 1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & -1 & 3 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \rightarrow$

from the echelon form $n = 5$, $r = 3$, there are $n - r = 2$ free variables, i.e. y and w. So, the general solution, if $y = \alpha$ and $w = \beta$, is $\begin{pmatrix} -2\alpha - 6\beta \\ \alpha \\ 2\beta \\ -\beta \\ \beta \end{pmatrix}$. For example, if $\alpha = 1$ and $\beta = 1$, one of the non trivial solutions is $\begin{pmatrix} -8 \\ 1 \\ 2 \\ -1 \\ 1 \end{pmatrix}$.

4. $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 4 & 4 \\ 3 & 7 & 10 & 14 \\ -1 & -3 & -2 & -6 \end{pmatrix} \rightarrow \boldsymbol{U} = \begin{pmatrix} 1 & 2 & 4 & 4 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and $\boldsymbol{L} = \boldsymbol{E}_{21(3)}\, \boldsymbol{E}_{31(-1)}\, \boldsymbol{E}_{32(-1)} =$

$$\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -1 & -1 & 1 \end{pmatrix} \rightarrow \boldsymbol{A} = \boldsymbol{LU}.$$

5. $\begin{cases} x_1 - 2x_2 + x_3 = 0 \\ 3x_1 + x_2 - 2x_3 = 9 \\ -2x_1 + 3x_2 - x_3 = -1 \end{cases} \rightarrow |\boldsymbol{A}| = \begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & -2 \\ -2 & 3 & -1 \end{vmatrix} = 2,\quad |\boldsymbol{A}1| = \begin{vmatrix} 0 & -2 & 1 \\ 9 & 1 & -2 \\ -1 & 3 & -1 \end{vmatrix} = 6,$

$|\boldsymbol{A}2| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 9 & -2 \\ -2 & -1 & -1 \end{vmatrix} = 4$, and $|\boldsymbol{A}3| = \begin{vmatrix} 1 & -2 & 0 \\ 3 & 1 & 9 \\ -2 & 3 & -1 \end{vmatrix} = 2$. Thus, $x_1 = \dfrac{|A1|}{|A|} = 3$,

$x_2 = \dfrac{|A2|}{|A|} = 2$, and $x_3 = \dfrac{|A3|}{|A|} = 1$.

6. Given $\boldsymbol{B} = \begin{pmatrix} -2 & 1 & 3 \\ 3 & -2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, one of the pairs $\boldsymbol{P}$ and $\boldsymbol{Q}$ is $\boldsymbol{P} = \begin{pmatrix} 0 & 1 & -2 \\ 0 & 1 & -3 \\ 1 & 1 & -1 \end{pmatrix}$ and

$Q = \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{pmatrix}$ such that $\boldsymbol{PBQ} = \boldsymbol{N} = \begin{pmatrix} I_2 & 0 \\ 0 & 0 \end{pmatrix}$.

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