Mid Semester Test November 2011

Course : Matrix Algebra

Department : Mathematics Education

Semester : I

Guidelines of the answers**:**

1. f(**A**) = 3**A**³ + **A**² – 2**A** + 3**I**

$$f(\boldsymbol{A}) = 3\begin{pmatrix} 22 & 14 & 0 \\ -21 & -13 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 10 & 6 & 0 \\ -9 & -5 & 0 \\ 0 & 0 & 1 \end{pmatrix} - 2\begin{pmatrix} 4 & 2 & 0 \\ -3 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + 3\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$f(\boldsymbol{A}) = \begin{pmatrix} 71 & 44 & 0 \\ -66 & -39 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

2. Given $\boldsymbol{A} = \begin{pmatrix} x & y \\ z & s \end{pmatrix}$. **A** is orthogonal matrix → $\boldsymbol{A}\boldsymbol{A}^T = \boldsymbol{I}$

$\begin{pmatrix} x & y \\ z & s \end{pmatrix}\begin{pmatrix} x & z \\ y & s \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, so the requirement for $x, y, s$, and $s$ are :

$x^2 + y^2 = 1$ (1)

$xz + ys = 0$ (2)

$z^2 + s^2 = 1$ (3)

for example, from equation (1) if $x = \frac{1}{2}$, then $y = \pm\frac{1}{2}\sqrt{3}$; and from equation (3) if $s = \frac{1}{2}$, then $z = \pm\frac{1}{2}\sqrt{3}$; so based on equation (2), it must be taken (for example) $x = \frac{1}{2}$, $y = -\frac{1}{2}\sqrt{3}$, $s = \frac{1}{2}$, and $z = \frac{1}{2}\sqrt{3}$ . thus the matrix $\boldsymbol{A} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$, or you can take the other combination which are satisfying the equation (1), (2), and (3) above.

3. Suppose $\boldsymbol{A} = \begin{pmatrix} -1 & x \\ y & 1 \end{pmatrix} \rightarrow \boldsymbol{A}\boldsymbol{A} = \boldsymbol{I}$

$$\begin{pmatrix} -1 & x \\ y & 1 \end{pmatrix}\begin{pmatrix} -1 & x \\ y & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$1 + xy = 1$

$xy + 1 = 1 \rightarrow xy = 0$

from that equation, $xy = 0$, it is mean that $x = 0$ or $y = 0$. Thus matrix $\boldsymbol{A} = \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}$, or $\begin{pmatrix} -1 & 0 \\ -7 & 1 \end{pmatrix}$, or $\begin{pmatrix} -1 & 0 \\ 5 & 1 \end{pmatrix}$, etc.

4. $|\boldsymbol{K}| = \begin{vmatrix} 1 & -3 & 4 & 2 \\ -4 & 7 & -2 & 5 \\ 0 & 6 & -3 & 4 \\ -2 & 5 & 2 & 3 \end{vmatrix} = 402.$

5. $\begin{vmatrix} 2 & -1 & 1 \\ x-5 & 0 & 1 \\ 4 & -5 & x+1 \end{vmatrix} = 12 \leftrightarrow x^2 - 9x + 14 = 0 \rightarrow x = 2$ or $x = 7$.

6. $\begin{vmatrix} a & b & c \\ a+d & b+e & c+f \\ a+d+g & b+e+h & c+f+i \end{vmatrix} \rightarrow$ apply : row 3 – row 2, then continued row 2 – row 1

become : $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$ .

7. (a) Given : $\boldsymbol{A}$ and $\boldsymbol{B}$ are symmetric matrices

$$\boldsymbol{A}^{\mathrm{T}} = \boldsymbol{A} \text{ and } \boldsymbol{B}^{\mathrm{T}} = \boldsymbol{B};$$

proved : $\boldsymbol{AB}$ is symmetric if and only if $\boldsymbol{A}$ and $\boldsymbol{B}$ are *commute*

$$(\boldsymbol{AB})^{\mathrm{T}} = \boldsymbol{AB} \leftrightarrow \boldsymbol{AB} = \boldsymbol{BA}$$

***Proof:*** (i) $(\boldsymbol{AB})^{\mathrm{T}} = \boldsymbol{AB}$

$\boldsymbol{B}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}} = \boldsymbol{AB}$ (properties of Transpose)

$\boldsymbol{AB} = \boldsymbol{BA} \rightarrow \boldsymbol{A}$ and $\boldsymbol{B}$ are commute

(ii) $\boldsymbol{AB} = \boldsymbol{BA}$

$\boldsymbol{AB} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}}$

$\boldsymbol{AB} = (\boldsymbol{AB})^{\mathrm{T}} \rightarrow \boldsymbol{AB}$ is symmetric.

(b). Given : $\boldsymbol{B}$ is *involuntary* $\rightarrow \boldsymbol{B}^2 = \boldsymbol{I}$.

show that : $(\boldsymbol{I} - \boldsymbol{B})$ is *idempotent* $\rightarrow (\boldsymbol{I} - \boldsymbol{B})^2 = (\boldsymbol{I} - \boldsymbol{B})$

proof: $(\boldsymbol{I} - \boldsymbol{B})^2 = (\boldsymbol{I} - \boldsymbol{B})(\boldsymbol{I} - \boldsymbol{B}) = \boldsymbol{I}^2 - 2\boldsymbol{BI} + \boldsymbol{B}^2 = 2(\boldsymbol{I} - \boldsymbol{B})$

---Good Luck---